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TOWARDS STANDARDIZATION IN TERMINAL BALLISTICS  
TESTING: VELOCITY REPRESENTATION

J. P. Lambert, et al

Ballistic Research Laboratories  
Aberdeen Proving Ground, Maryland

January 1976

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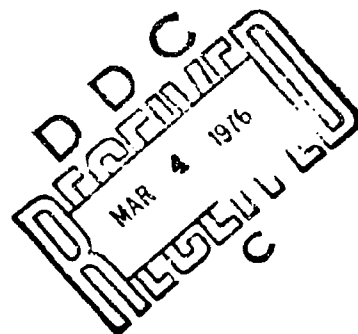
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J. P. Lambert  
G. H. Jonas

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  An important phase of a prospective general procedural standardization of ballistic velocity testing and evaluation is considered. To accommodate relationships between striking and residual velocities, a basic form is 'developed' from a natural generalization of existing theory and a numerical scheme for parameter determination is presented. Actual evaluation of relevant form parameters from given ( $v_s$ , $v_r$ ) data sets is afforded by an associated Fortran program which additionally incorporates a plot routine to provide graphic representation.		

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## I. INTRODUCTION

Many aspects of terminal ballistics testing are in apparent need of procedural standardization. We attempt here to deal in part with one such area: representation of the relationship between striking and residual velocities implicit in sets of  $(v_s, v_r)$  data.

A form is proposed as being sufficiently simple and versatile to usefully and realistically model velocity interdependence. A regression technique (a direct non-linear least squares algorithm) is presented which, through empirical parameter determination, establishes a systematic method for obtaining an explicit functional relationship; in particular, a "limit velocity" is thereby routinely generated from input data. It is suggested that what is here described might substantially contribute towards a standardized methodology for assessment and categorization of velocity data and towards standardization of strategy in testing.

In short, if the overall procedure is accepted, then a formula is at hand to concisely contain experimental velocity information. Indeed much of the information from a data set is condensed into a triple  $(a, v_L, p)$  of numbers which uniquely specifies the relationship. Ideally, and in the nature of future intent, these numbers should be qualified according to data variance and endowed with a statistical appraisal of confidence.

The practicable essence of this report is cast in a comprehensive list of Fortran directives which methodically induces machines to extract parameters from data and to plot corresponding  $(v_s, v_r)$  curves.

## II. A PROPOSED FORM

### 1. Context and Form Considerations

A frequent test, of multiple special interest, in terminal ballistics consists of the firing of a number of nominally identical projectiles (penetrators) into as many nominally identical targets with all controlled phenomena except for striking velocity\*,  $v_s$ , being nominally invariant. The measured response to each such impact which is of present concern is projectile residual velocity,  $v_r$ . A collection of data points  $(v_s, v_r)$  is thereby obtained. In accord with common usage, each shot is deemed to result in "penetration," while "perforation" is signalled by the criterion  $v_r > 0$ .

---

\*For present purposes we regard striking velocity as a controlled variable. Also, in deference to prevailing custom, velocities are taken to be non-negative numbers rather than vectors.



Let us suppose a given projectile-target situation, by which it is assumed that a given projectile impacts a given target with variable striking velocity  $v_s$  but with all other relevant pre-impact characteristics being given and constant. From this point and this premise the discussion admits of mathematical idealization. In the given context residual velocity can be considered a function of striking velocity;  $v_r = f(v_s)$  for  $v_s > 0$ . We assume the existence of a limit velocity,  $v_l$ , characterized (i.e., defined) by the properties:  $v_r = 0$  for  $0 \leq v_s \leq v_l$  and  $v_r > 0$  for  $v_s > v_l$ ; equivalently,  $v_l = \max \{v_s : v_r = 0\} = \inf \{v_s : v_r > 0\}$ .\*

As further characteristics of a suitable (and workable) model, we desire continuity and that, for  $v_s > v_l$ , the function be strictly increasing, smooth (differentiable) and concave\*\* (every chord lies beneath its intercepted arc). The latter properties are secured by requiring that the function possess at each point on its support ( $\{x : f(x) \neq 0\}$ ) a positive first and a negative second derivative. Such stipulation is of course largely contrivance and is not experimentally decidable but is convenient, suggestive of physical experience, and imposing of no apparent conflict with experiment or theory.

## 2. Further Form Considerations: Simple Penetration Theory and the Hyperbola

Traditional penetration theory, especially where an explicit model is developed, has been most extensive (and evasive of much unpleasantness) in dealing with the case of a rigid (mass-preserving), essentially non-deforming penetrator; e.g., a steel projectile impacting a thin aluminum plate. Such treatments, otherwise tolerating a diversity of assumptions about the physics of the particular penetration process conjectured, have been numerous.

It is a pertinent observation, and is perhaps not generally recognized, that the penetration models evolved from these theories have almost invariably adhered to one basic form; specifically:

$$v_r = \begin{cases} 0, & 0 \leq v_s \leq v_l \\ a(v_s^2 - v_l^2)^{1/2}, & v_s > v_l \end{cases} \quad (1)$$

\* Here and elsewhere we specifically exclude the possibility of modeling a "shatter regime," i.e., we are insisting that if  $f(x_1) > 0$  and  $x_2 > x_1$ , then  $f(x_2) > 0$ . This is not considered a serious compromise with the generality for which we strive.

The infimum of a set is its greatest lower bound.

\*\* This precludes the possibility of inflection points.

For instance, the Poncelet-Morin hypothesis\* of 1829 concerning the resistance offered a projectile in motion through a dense medium leads, with supplementary assumptions, rather easily to this form (c.f. Robertson [1943]); so also does the quite different simple energy-momentum analysis of Recht and Ipson [1963]. A variety of other approaches falls into line (Nishiwaki, Thomson, Zaid and Paul, etc.). The various models ultimately differ precisely in so far as do the formulations for  $\alpha$  and  $v_\ell$ ; which quantities we will tend to view as parameters to be subjected to optimal adjustment in a given situation.

Our examination of available data tends to confirm that experimental results can often be well represented within the framework of form (1), particularly in situations where there is not (or is not expected to be) excessive projectile deformation. Theory and experience persuade us that in many instances (1) does offer a viable model. We will feel obliged presently to adopt many characteristics of this form, and the form itself as a special case, in a proposed more versatile basic form.

We note that form (1) meets the requisites of the previous section. Additionally, there is an infinite right derivate at  $v_\ell$ ; i.e., while

$\frac{d v_r}{d v_s}$  does not exist at  $v_\ell$ , it is true that  $\lim_{v_s \downarrow v_\ell} \frac{d v_r}{d v_s} = \infty$ . Also  $\alpha v_s$  approximates  $v_r$  for large values of  $v_s$ ; precisely,  $\lim_{v_s \rightarrow \infty} (\alpha v_s - v_r) = 0$ .

Observe that for  $v_s \geq v_\ell$  (1) is a quadrant of a hyperbola with center at the origin, major axis coincident with the horizontal axis, having as one asymptote the line through the origin of slope  $\alpha$ .

The assumption of projectile rigidity typically associated with this form facilitates dealing in terms of kinetic energy; within the perforation regime, (1) implies linearity between striking and residual energies:  $e_r = \alpha^2 (e_s - e_\ell)$ , where symbol meanings are presumably evident. It may at this point be worth noting that even in cases where significant deformation and mass loss are experienced, we have observed tentative experimental indication (from situations where residual mass measurements are available) of approximate linear correlation between striking and residual energies.

---

\* A modification of earlier "sectional-pressure" theories asserting that the force resisting penetration at a given target depth is jointly proportional to a depth-dependent cross-sectional projectile area and to a linear function of instantaneous projectile energy.

Since  $v_r$  cannot exceed  $v_s$ , an imposed parameter constraint will be  $0 \leq \alpha \leq 1$ . In the event of perforation by a rigid projectile, an implication of the special case  $\alpha = 1$  is that energy lost to the penetration process is independent of striking velocity. The various proposed models of type (1) generally concur that  $\alpha$  should be near one for thin targets and, other things being equal, should decrease with increasing target thickness. For example, from the formulation of Recht and Ipsen for an assumed plugging mode of failure, we have  $\alpha = (1 + r)^{-1}$  where  $r$  is the ratio of the mass of the ejected plug (small for thin plates) to that of the projectile.

Otherwise extant simple penetration models are prone towards abject empiricism - randomly selected theories alleged to fit experimental data, limited of scope, curiously behaved, and in any event not sufficiently adaptable for our purposes to be usefully exploited\*.

### 3. Present Proposal: The Basic Form

There is clear experimental indication that in general a somewhat more flexible model is required to represent the velocity relationship being considered. In particular it has been found that form (1) is often not capable of adequately reflecting observed behavior for  $v_s$  above and in the vicinity of the derived  $v_0$ ; such deficiency can be rather dramatically apparent for instance in multiple plate situations and in the case of long rods perforating comparatively thick armor. Nor is this especially surprising for it is just in this region that one might suppose deformation to be most consequential; it is expected, c.f. Defourneaux [1973], that the proportional contribution of plastic deformation (and of friction) to total energy expenditure diminishes towards zero as  $v_s$  becomes large. In addition to the vagaries introduced by the possibility of projectile deformation, the general inadequacy of (1) might be reflected as well in variability of target deformation. More significant than the absolute susceptibility to deformation of either projectile or target, we expect, may be the manner in which the deformation patterns are distributed with striking velocity.

A basic form assimilating desired general characteristics\*\* and considered, from a wide spectrum of experimental evidence, well suited to effectively represent observed behavior is:

---

\* Some frequently encountered phenomena of these models which we find restrictive or at variance with our physical expectations are unaccountable discontinuities and inflection points, either lack of an asymptote or insistence that the asymptote have always a slope of one (or some other immutable value chosen by caprice) and absence of an infinite right slope (derivative) at  $v_0$ .

\*\* A partial exception will be noted in the next section.

$$v_r = \begin{cases} 0, & 0 \leq v_s \leq v_l \\ \alpha (v_s^p - v_l^p)^{1/p}, & v_s > v_l \end{cases} \quad (2)$$

with constraints  $0 \leq \alpha \leq 1$  and  $p > 1^*$ .

It is regarded that (2) offers additional and sufficient versatility precisely in the region where (1), which is a special case of (2), was found to be deficient. The parameter  $\alpha$  assumes its previous role and is most visible as the slope of the asymptote; analogous asymptotic behavior is insured by the requirement  $p > 1$ . The parameter  $p$ , sensitive primarily it is felt to deformation, we view as a shape factor controlling how sharply the function rises towards the asymptote\*\*. The injection of the variable  $p$  in place of 2 is inescapably empirical but procures an especially natural and appropriately conservative generalization. Both  $\alpha$  and  $p$  are clearly devoid of physical dimension. It is possible, though not at issue here, that for a large class of situations the three parameters can be sufficiently well prescribed in terms of relevant physical dimensions so as to determine an explicit functional model of predictive scope (one would for instance selectively borrow from rigid penetrator theory for  $\alpha$  and  $v_l$  formulation) and this theme may be pursued later.

It is rather our present purpose to advocate that form (2) serve as a standard framework within which to cast experimental results; we further propose that explicit relationships be obtained from given sets of  $(v_s, v_r)$  data by the systematic determination of triples  $(\alpha, v_l, p)$  of parameters from the regression procedure which is to follow (or, more literally, from the associated computer program).

The parameter  $v_l$ , marking the boundary of perforation, is often of special import. Standardized adoption of the outlined scheme would include acceptance of a  $v_l$  so generated as being definitively the limit velocity (for the given projectile-target situation) implicit in the given data set.

\* 1. There is resemblance in form to what have been called "power means,"

expressions of the form  $M_p = \left( \frac{x^p + y^p}{2} \right)^{1/p}$ .  $M_{1/3}$  for example is the Lorentz mean in the theory of equation of state for gases.

2. It is both interesting and irrelevant to recall Fermat's notorious "last theorem" (17th Century), asserting that for  $a = 1$ ,  $v_s > v_l > 0$  and  $p > 2$ , there are no integral values for the symbols  $v_s$ ,  $v_l$  and  $p$  which satisfy (2).

\*\* While it is of course true that  $\alpha v_s - v_r \rightarrow 0$  as  $v_s \rightarrow \infty$  for fixed  $p > 1$  (i.e., there is an asymptote of slope  $\alpha$  emanating from the origin), it is also true that  $\alpha v_s - v_r \rightarrow 0$  as  $p \rightarrow \infty$  for fixed  $v_s > v_l$ .

#### 4. Comments, Perspectives, and Implications

(i) Initial expectation was that a non-deforming projectile represented an ideal, that (mindful of traditional theory) such perfection could be best accommodated for  $p = 2$ , and that aberrations should if anything require higher values of  $p$  (roughly, that increase in deformation should correspond to increased rate of ascent of the  $(v_s, v_r)$  curve - that the "ideal" should correspond to a limiting value of the parameter  $p$ ). Two seemed a likely minimum. Whatever the merits of the reasoning, the expectation has, in our experience, been borne out for situations involving long rod penetrators, but not in general; fragments provide a frequent exception.

(ii) As there is occasional concern in the literature and elsewhere with change or loss of projectile velocity effected by penetration, it might be of some value to consider form (2) from this perspective. Letting  $\Delta = v_s - v_r$  we rewrite (2) as:

$$\Delta = \begin{cases} v_s, & 0 \leq v_s \leq v_\ell \\ v_s - \alpha (v_s^p - v_\ell^p)^{1/p}, & v_s > v_\ell \end{cases} \quad (3)$$

It is clear that the corresponding curve is asymptotic to the line from the origin of slope  $1 - \alpha$ ;  $\Delta$  is thus obliged to ultimately, with increasing striking velocity, either decrease towards zero or increase without bound depending on whether  $\alpha$  is one or less.

For  $v_s > v_\ell$  and  $\alpha < 1$  it is routinely verified that  $\Delta$  achieves a minimum value (and the corresponding point on the  $(v_s, v_r)$  curve has

slope 1) at the point  $v_s = v_\ell (1 - \alpha^{\frac{p}{p-1}})^{-1/p}$ . Experimental affirmation that velocity loss can, as implied, decrease to a relative minimum and then increase is noted e.g., by Goldsmith and Finnegan [1973].

(iii) Another perspective can be attained by considering transformations which map the region under the  $(v_s, v_r)$  curve into finite area. We speculate that such device may be of use in providing additional measure of parameter effect. As an example we offer one such transformation which maps functions of type (2) onto the unit interval and yields area as a function of  $p$  while normalizing the effect of  $\alpha$  and  $v_\ell$ .

Let  $w = \frac{v_r}{\alpha v_s}$  and  $z = \frac{v_\ell}{v_s}$ . For  $v_s > v_\ell$  we rewrite (2) as:

$$w = (1 - z^p)^{1/p} \quad (4)$$

The associated curve is concave, symmetric about the line  $w = z$  and contained by the triangle with vertices  $(0, 1)$ ,  $(1, 0)$  and  $(1, 1)$ ; for  $p = 2$  the curve is a quadrant of the unit circle. Area under the curve,

$\int_0^1 (1 - z^p)^{1/p} dz$ , is an increasing function of  $p$  with limiting values of  $1/2$  and  $1$  for  $1 < p < \infty$ .

An analogous measure, and one more computationally immediate, is provided by the  $L_p$  norm. Letting  $g(z) = w$  we observe that  $g$  is an element of the (classical Banach) space  $L_p(0, 1)^*$ . The appropriate norm is given by

$$\|g\|_p = \left( \int_0^1 |g|^p \right)^{1/p} = \left( \int_0^1 (1 - z^p) dz \right)^{1/p} = \left( \frac{p}{p+1} \right)^{1/p}.$$

$\|g\|_p$  is similarly an increasing function of  $p$ , and  $1/2 < \|g\|_p < 1$  for  $1 < p < \infty$ .

(iv) This section is concluded by noting an appealing property of form (1) which is not preserved in the generalization to form (2).

The composition of two functions of form (1) is itself of this form\*\* (composition is a binary operation on the class of functions of this form). If (1) is regarded as the operable medium and a situation involves a target comprised of several parallel plates, then (1) can be used consistently for both the whole target situation and that of each "subtarget." In particular the specifying parameters for the target can be calculated from those for the constituent plates.

Unhappily, such is not the case within the context of form (2) unless  $p$  is set at some fixed value. To be quite specific: suppose we are dealing with a target ensemble of two plates perforated by a

\* i.e.,  $g$  is Lebesgue measurable on  $(0, 1)$  and  $\int_0^1 |g|^p < \infty$ . It is of course equally true that  $g \in L_q(0, 1)$  for all  $1 \leq q < \infty$  but in the case of  $p$  the observation is especially natural.

\*\* Basically, because the class of appropriately oriented hyperbolas is closed under composition; if  $f_1(x) = a \sqrt{x^2 - v^2}$  and  $f_2(x) = b \sqrt{x^2 - w^2}$

then  $f_1(f_2(x)) = ab \sqrt{x^2 - (v^2 + \frac{w^2}{a^2})}$ : the composition  $f_1 f_2$  is a hyperbola of the same type as  $f_1$  and  $f_2$ . (To go further, the class of such hyperbolas is an abelian group under the composition operation).

projectile having initial, between-plate and final velocities denoted by  $x$ ,  $y$  and  $z$  respectively. Then we would like to say that  $y = a (x^p - v^p)^{1/p}$  and  $z = b (y^q - w^q)^{1/q}$  for some  $a$ ,  $b$ ,  $v$ ,  $w$ ,  $p$  and  $q$ . But the implied overall striking-residual connection is then

$z = ab [(x^p - v^p)^{q/p} - (w/a)^q]^{1/q}$  which does not conform to (2). This lack of closure under composition, while not strictly an impairment to the present concern of supplying effective representations of velocity relationships, does preclude full mathematical consistency in the consideration of multiple plate targets.

### III. THE PARAMETERS

#### 1. A Regression Procedure for Parameter Estimation

We now suggest a mechanism for the determination of explicit form (2) representations from given data sets. It is convenient for the moment to replace  $v_s$ ,  $v_r$  and  $v_\ell$  in (2) with  $x$ ,  $y$  and  $z$  respectively.

Following is a least squares algorithm for fitting the form

$$y = \begin{cases} 0, & 0 \leq x \leq c \\ a (x^p - c^p)^{1/p}, & x > c \end{cases}$$

with constraints  $0 \leq a \leq 1$  and  $p > 1$  to a data set  $\{(x_1, y_1), \dots, (x_n, y_n)\}$  of pairs of non-negative integers, each  $x_i$  and at least one  $y_i$  being positive. It is assumed that  $n \geq 2$ ,  $x_1 < x_n$ , and  $x_1 \leq x_2 \leq \dots \leq x_n$ .<sup>\*</sup> Parameters subjected to estimation are  $a$ ,  $p$  and  $c$  (or otherwise as indicated).

For the mechanics of the algorithm we will also suppose that  $p \leq 8$ ; this is a harmless restriction as there is but minimal variation in the form for  $p > 8$  ( $a$  and  $c$  fixed)<sup>\*\*</sup>, and it is necessary to specify some upper bound for  $p$ . Further, reflecting upon the input data and the interval spacing selected for search routines,  $c$  will be required to be an integer and  $p$  will be confined to the set  $\{1.1, 1.2, \dots, 8\}$ . These

<sup>\*</sup>That is, we need at least two data pairs and at least two distinct  $x$  values; additionally, the data pairs are assumed to be ordered according to increasing values of  $x$  — this latter requirement will in practice be obviated by the inclusion of a preliminary sorting routine in the computer program.

<sup>\*\*</sup>A measure of this variation is conveniently afforded by the  $L_p$  norm specified in II 4 (iii). We note that  $\|e\|_8 > .985$  while  $\lim_{p \rightarrow \infty} \|e\|_p = 1$ .

strictures have been found reasonable for the particular application in mind but are of course easily alterable.

Immediate interest is in what will be called the standard program, SP, in which the complete parameter set  $(\alpha, c, p)$  is estimated. There is related concern with the following special options which it is convenient to simultaneously establish;

$0_1$ :  $p$  is prescribed, only  $\alpha$  and  $c$  are determined,

$0_2$ :  $\alpha$  is prescribed, only  $c$  and  $p$  are determined,

$0_3$ :  $\alpha$  and  $p$  are prescribed, only  $c$  is determined.

In the usual spirit of the least squares approach, we seek in each case appropriately constrained parameters which approximately minimize the function

$$\psi(\alpha, c, p) = \sum_{x_i < c} y_i^2 + \sum_{x_i \geq c} [\alpha (x_i^p - c^p)^{1/p} - y_i]^2.$$

Root mean square error (or standard error) is expressed by  $\sqrt{\frac{1}{n} \psi(\alpha, c, p)}$ .

With no possible loss in precision (as measured by the value of  $\psi$ ), we require that  $0 \leq c \leq x_n$  and, for SP and  $0_1$ , that  $\alpha = 1$  if  $c = x_n$ .

$0_1$  and  $0_3$

$$\text{Let } g(c) = \sum_{x_i \geq c} y_i (x_i^p - c^p)^{1/p} \text{ and } h(c) = \sum_{x_i \geq c} (x_i^p - c^p)^{2/p}, \quad c=0, 1, \dots, x_n.$$

Then  $g \geq 0$  and  $h \geq 0$  for  $0 \leq c < x_n$ ;  $g(x_n) = h(x_n) = 0$ ; and

$$\psi(\alpha, c, p) = \sum_{i=1}^n y_i^2 - \alpha (2g - ah).$$

In the case of  $0_1$ , if  $0 \leq c < x_n$ ,  $\psi$  is a quadratic in  $\alpha$  having an absolute minimum at  $\alpha = g/h \geq 0$   $\left( \frac{\partial \psi}{\partial \alpha} = 0, \frac{\partial^2 \psi}{\partial \alpha^2} > 0 \right)$ . For fixed  $c$ ,  $\psi$  is thus minimized (exactly) for  $0 \leq \alpha \leq 1$  by taking  $\alpha = \min(g/h, 1)$ .



Accordingly, the optimal value for  $a$  corresponding to a given value  $c$  is

$$\hat{a}(c) = \begin{cases} g(c)/h(c), & g(c) < h(c) \\ 1, & g(c) \geq h(c). \end{cases}$$

For  $0_1$  and  $0_3$  respectively, define:

$$(i) \quad \phi(c) = \hat{a}(c) [2g(c) - \hat{a}(c)h(c)] = \begin{cases} g^2(c)/h(c), & g(c) < h(c) \\ 2g(c) - h(c), & g(c) \geq h(c) \end{cases}$$

and

$$(ii) \quad \phi(c) = a [2g(c) - ah(c)].$$

In both cases the expression  $\sum_{i=1}^n y_i^2 - \phi(c)$  needs to be minimized; this is accomplished by finding a value  $\hat{c}$  which (approximately) maximizes the function  $\phi(c)$ .

We first generate an initial estimate  $c_0$ .

Let  $\alpha = \min \{i = 1, 2, \dots, n : y_i > 0\}$ ,  $x_0 = y_0 = 0$ , and

$\beta = \max \{i = 0, 1, \dots, n : y_i = 0\}$ .

$c_0$  then derives from a linear least squares fit of  $y^P = a^P(x^P - c^P)$  on  $\{(x_i^P, y_i^P) : i \geq \alpha\}$  with constraints  $c_0 \geq 0$ , and  $c_0 \geq x_\beta$  if  $\alpha > \beta$ .\*

The explicit procedure for generating  $c_0$  is as follows:

Let  $r = n - \alpha + 1$ ,

$$b_1 = \sum_{i=\alpha}^n x_i^{2P}, \quad b_2 = \sum_{i=\alpha}^n x_i^P, \quad b_3 = \sum_{i=\alpha}^n y_i^P, \quad b_4 = \sum_{i=\alpha}^n (x_i y_i)^P,$$

---

\*In the language of our physical model, we are constraining the initial limit velocity estimate to be at least as large as the largest striking velocity for which there is no perforation unless perforation does occur at some smaller striking velocity.

Note that the linear regression does not use data points to the left of the first point for which there is perforation.

$$u = \begin{cases} 0, & rb_1 = b_2^2 \\ \frac{rb_4 - b_2 b_3}{rb_1 - b_2^2}, & rb_1 \neq b_2^2 \end{cases} \quad \text{and, to the nearest integer,}$$

$$t = \begin{cases} \left( \frac{b_2 u - b_3}{ru} \right)^{1/r}, & u > 0 \text{ and } b_2 u > b_3 \\ 0, & \text{otherwise.} \end{cases}$$

$$\text{Then let } c_0 = \begin{cases} t, & \alpha \leq \beta \\ \max(t, x_\beta), & \alpha > \beta \end{cases}.$$

Normally  $c_0$  should be a very good estimate for the parameter  $c$  and, as a simplified method suitable for desk calculators, it might be acceptable to stop at this point; it should then however be realized that  $c$  has not been derived from a direct non-linear least squares fit of the form to the data but results rather from a linear fit to a transformation of a subset of the data.

The last step is to search for a better estimate.

Routine 1: Starting from  $c = c_0$ , successively evaluate  $\phi(c)$ , letting  $c$  vary by steps of one unit in the direction of increasing  $\phi$  (if there should be a local minimum at  $c_0$ , proceed to  $c_0+1$ ); let  $\hat{c}$  be the first value for  $c$  such that  $\phi(c)$  is at least as large as both  $\phi(c-1)$  and  $\phi(c+1)$ , or such that  $c = 0$ . Formally: let  $\hat{c} = c_0 + j_0$ , where

$$j_0 = 0 \text{ if } \phi(c_0) \geq \phi(c_0+1) \text{ and either } c_0 = 0 \text{ or } \phi(c_0) \geq \phi(c_0-1),$$

$$j_0 = \min \{j \geq 1 : \phi(c_0+j+1) \leq \phi(c_0+j)\} \text{ if } \phi(c_0) < \phi(c_0+1),$$

$$j_0 = -\min \{j \geq 1 : \phi(c_0-j-1) < \phi(c_0-j) \text{ or } j = c_0\} \text{ if}$$

$$\phi(c_0+1) \leq \phi(c_0) < \phi(c_0-1).$$

While there is no assurance that  $\hat{c}$  provides the largest possible value for  $\phi(c)$ , it does better than the initial estimate,  $c_0$ , which is itself expected to be good and further searching is considered unwarranted.

For  $O_1$  the parameter estimates are  $a = \hat{a}(\hat{c})$  and  $c = \hat{c}$ .

For  $O_3$  the estimated parameter is  $c = \hat{c}$ .

## SP and $O_2$

We rely upon much of the preceding technique while allowing  $p$  to vary.

For a fixed value of  $p$ , let  $g(c)$ ,  $h(c)$  and  $\hat{a}(c)$  be as before and let  $\phi(c)$  be defined as in (i) and in (ii) for the cases SP and  $O_2$  respectively.

For  $p = 2$ , determine  $c_0$  as described previously; then use Routine 1 to find the corresponding  $\hat{c}$ , which we now denote as  $\hat{c}(2)$ .

For  $p = 2.1, 2.0, \dots, 1.1$  respectively, let  $\hat{c}(p)$  be obtained from its predecessor by using Routine 1 with  $c_0$  replaced by  $\hat{c}(p - .1)$ . Similarly, for  $p = 1.9, 1.8, \dots, 1.1$ , let  $\hat{c}(p)$  be successively generated by using Routine 1 with  $c_0$  replaced by  $\hat{c}(p + .1)$ .

Let  $\hat{\phi}(p) = \phi(\hat{c}(p))$ ,  $p = 1.1, 1.2, \dots, 2.0$ .

Approximate maximization of  $\hat{\phi}(p)$  is desired and we follow a search routine, starting from  $p = 2$ .<sup>\*</sup> Explicitly: let  $\hat{p} = 2 + k/10$  where  $k = \min \{j = 0, 1, \dots, 60 : \hat{\phi}(2+j/10) \geq \hat{\phi}(2.1+j/10) \text{ or } j = 60\}$  if  $\hat{\phi}(2) \leq \hat{\phi}(2.1)$ ;

$k = - \min \{j = 0, 1, \dots, 9 : \hat{\phi}(2-j/10) \geq \hat{\phi}(1.9-j/10) \text{ or } j = 9\}$   
if  $\hat{\phi}(2) > \hat{\phi}(2.1)$ .

For SP the parameter estimates are  $a = \hat{a}(\hat{c}(\hat{p}))$ ,  $c = \hat{c}(\hat{p})$  and  $p = \hat{p}$ .

For  $O_2$  the estimates are  $c = \hat{c}(\hat{p})$  and  $p = \hat{p}$ .

## 2. A Remark

Reliance upon a direct non-linear regression technique necessitates high-speed computing ability. The form does not seem to lend itself to linearization in any acceptable manner but we do at least expect to

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<sup>\*</sup> Starting from  $p = 2$  is primarily a matter of computational expedience; we expect that in many instances the optimal value of  $p$  will be near two.

The search works away from two in the direction of increasing  $\hat{\phi}$  and stops when a local maximum is detected. There is no general guarantee of having hit upon the absolute maximum; however we have tried more extensive routines on a large and diverse class of data sets and have by comparison found the simple (and efficient) routine given here to be consistently satisfactory (typically the function  $\hat{\phi}$ , for the given data, had but one maximum.) We are confident that this routine does, for the intended application, provide a good estimate.

benefit in terms of accuracy from a direct approach. Prevailing against linearization are the nonlinearity of the form in the parameter  $p$  and the special role of the parameter  $v_\ell$  (or  $c$ ).

If one were content to take form (1) (or (2) with  $p$  prescribed), restrict this form to its support, delete certain data points and transform the remainder, then linearization is feasible. A problem here, besides loss of generality and incomplete utilization of information, is that a sound decision on point deletion can be critical and is not always apparent. An obvious but not consistently judicious choice is to delete all points for which there is no perforation; i.e., the linear form  $v_r^2 = a^2 (v_s^2 - v_\ell^2)$  is adapted to those points  $(v_s^2, v_r^2)$  for which  $v_r$  is positive. Such procedure is recurrent in the literature and goes back at least to Robertson [1943]. A rather more sophisticated approach along these lines was used for generating initial estimates in the preceding algorithm.

For the price of nonlinearity and a higher level of machine dependence we find it far preferable to systematically fit a complete and more general form to a complete data set.

### 3. Some Statistical Contingencies

The effort here is to suggest some plausible areas for statistical exploration. The questions raised will likely require better formulation from keener perspectives at another time and we will not at this point be particularly inhibited by lack of precision.

Till now we have adopted a deterministic view of velocity dependence. We have assumed the pretense of being able to deal with a specific residual velocity consequent to a given striking velocity; more realistically we should perhaps enquire of the probability that for a given striking velocity the residual velocity (regarded as a random variable) will belong to a specified interval. It is transparent that for each  $v_s$ ,  $\text{Prob } [0 \leq v_r \leq v_s] = 1$ ; \* significant statements of this nature would however be welcome.

To what extent can meaningful assessments of confidence be attributed to estimated parameters? Size of the data set is clearly important. Also significant surely, and in different ways for the different parameters, is the distribution of striking velocities; a concentration of points about the estimate  $v_\ell$  is bound to enhance confidence in that estimate but reflects little on the parameter  $a$  which is mostly influenced by points with high striking velocity. Estimation and exploitation of relevant variance measures is probably essential.

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\* Unless fortuity in measurement is considered to admit of the possibility that  $v_r > v_s$ .

The parameter  $v_L$  is often of primary interest. One would ideally hope that some reasonable assumption about probability distribution, interacting with statistical invention, could be brought to assert that for a reasonable data set the quantities  $\text{Prob}[v_r = 0 \mid v_s < v_L - \epsilon]$  and  $\text{Prob}[v_r > 0 \mid v_s > v_L + \epsilon]^*$  are reasonably large (proximate to one) for reasonably small positive values of  $\epsilon$ . It would be additionally provident were the above probabilities to equal one-half for  $\epsilon = 0$ , in which case  $v_L$  would be equivalent to what has elsewhere been termed " $v_{50}$ ", the concept of which, along with other probabilistic phenomena, has provoked the perpetration of much nonsense in ballistic literature.

Of allied interest would be a measure of relative confidence in different  $v_L$ 's extracted from different data sets. Consider the following situation. A collection of  $v_L$ 's is at hand and one is equipped with a general form purportedly able, over some broad range of situations including those which yielded the  $v_L$ 's, to represent limit velocity in terms of the physical set-up (materials, geometry, etc.); e.g., variants of the de Marre form\*\* have been fashionable in this regard for nearly a century. One then regresses to the collected  $v_L$ 's to evaluate parameters - but ideally the various  $v_L$ 's should be variously weighted in the relevant regression function so as to reflect their various relative degrees of reliability.

Another problem involves the design of methodology for data acquisition relative to a given projectile-target situation. With reference to *a priori* value judgments about information desired, and constrained by economic and physical limits, a suitable and efficient experimental strategy needs to be ordained. A new dimension of complexity is injected at this stage by the introduction of another random variable; striking velocities can, we suppose, be regarded as deterministically measured, but they certainly cannot be so controlled.

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\*  $\text{Prob}[A|B]$  is the (conditional) probability measure of A given B.

\*\* The most prevalent de Marre-type form and one in current local usage, a dimensionally purified revision of the original, can be written as  $v_L = (t/d)^\gamma \sqrt{kd^3/m}$  where  $d$  and  $m$  are projectile diameter and mass, and  $t$  is target thickness (or thickness times a function, such as secant, of incidence angle).  $k$ , with units of force per area (e.g. a multiple of yield stress), and  $\gamma$  are the parameters to be estimated. A (linearized) regression procedure in this case is apparent. The form is (perhaps obviously) empirical but not without an element of physical appeal, especially in the rigid-projectile context for which it was originally intended; indeed some quite serious theoretical analyses, invoking principles of elasticity and hydrodynamics, have produced models of this type with values for  $\gamma$  of 1/2 and 1 (our data-dependent least squares determinations for  $\gamma$  have typically been near .8).

If limit velocity is the sole objective, the attempt should be towards obtaining perforations and non-perforations within a small range of striking velocities.\* Should the intent be for uniformity of confidence in the full relationship, or if there is a weighted concern with perhaps disproportionate interest in limit velocity, then an imposing variety of concepts needs to be formalized and an appropriate experiment designed. At issue, briefly, is the prescription of standard procedures for generating sets of  $(v_s, v_r)$  data from which limit velocities and overall velocity correlation can, with measurable reliability, be systematically and definitively determined.

#### IV. THE COMPUTER PROGRAM

##### 1. Introduction and Examples of Output

The algorithm of Section III. 1. is encoded in a Fortran Program to generate parameter estimates from a given data set. Plot routines provide separate graphic displays of each of the consequent form (2), (3) and (4) representations; the form (2) representation (the  $(v_s, v_r)$  curve) being no doubt of predominate interest. In each case the given data set or the appropriate transformation thereof is also graphed; for (2) and (3) relevant asymptotes are plotted as dotted lines. In deference to machine notational limitations, the symbols  $v_s, v_r, v_\ell, a$  and  $p$  are replaced by  $X, Y, C, A$  and  $P$  respectively in plots and print-out.  $S$  will designate the root mean square error (c.f. page 17) associated with the fit of the determined  $(v_s, v_r)$  curve to the given  $(v_s, v_r)$  data set.

Although our special concern in this report is with the standard program SP (in which all three parameters are determined), we provide as well for the options  $O_1, O_2$ , and  $O_3$  described earlier and additionally for the possibility of plotting graphs and relevant points when all three parameters are initially prescribed.

One page of computer printout includes a tabular listing of the data set (observed striking and residual velocities) along with some derived quantities which may be of peripheral interest (e.g.,

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*\*One conjectures that a  $v_\ell$  so generated is inherently "better" than the analogous extraction from Sensitivity Analysis, in which there is sensitivity only to quantal response (yes or no as regards perforation) and which, provided residual velocities are watched, seems far less exploitative of information.*

the "error" at each observed striking velocity between the observed residual velocity and the residual velocity associated with the determined fit).

For SP and  $O_2$ , a second page of printout tabulates "optimal" values for A and C corresponding to each value for P considered by the search routine described on page 20; the root mean square error S associated with each such P is also calculated and printed. In particular, the "optimal" form (1) fit is determined (recall that the search routine in question starts from, and hence always examines, the case  $P=2$ ).

In the following pages we offer examples of machine output as generated by the standard program. These are meant to be more illustrative than substantive; actual projectile-target situations involved, though partially and cryptically specified in titles, are of little concern and we do not elaborate.

Figure 1 provides displays of striking versus residual velocity (form (2) representations) generated by two different data sets. Figure 2 depicts striking velocity versus residual velocity and striking velocity versus velocity loss (form (2) and (3) representations respectively) associated with the same data set. Next is an example of the complete output (in actual output size) from a single set of input data: tables 1a and 1b comprise the computer printout; figures 3a, 3b and 3c are respectively the forms (2), (3) and (4) plotter displays.

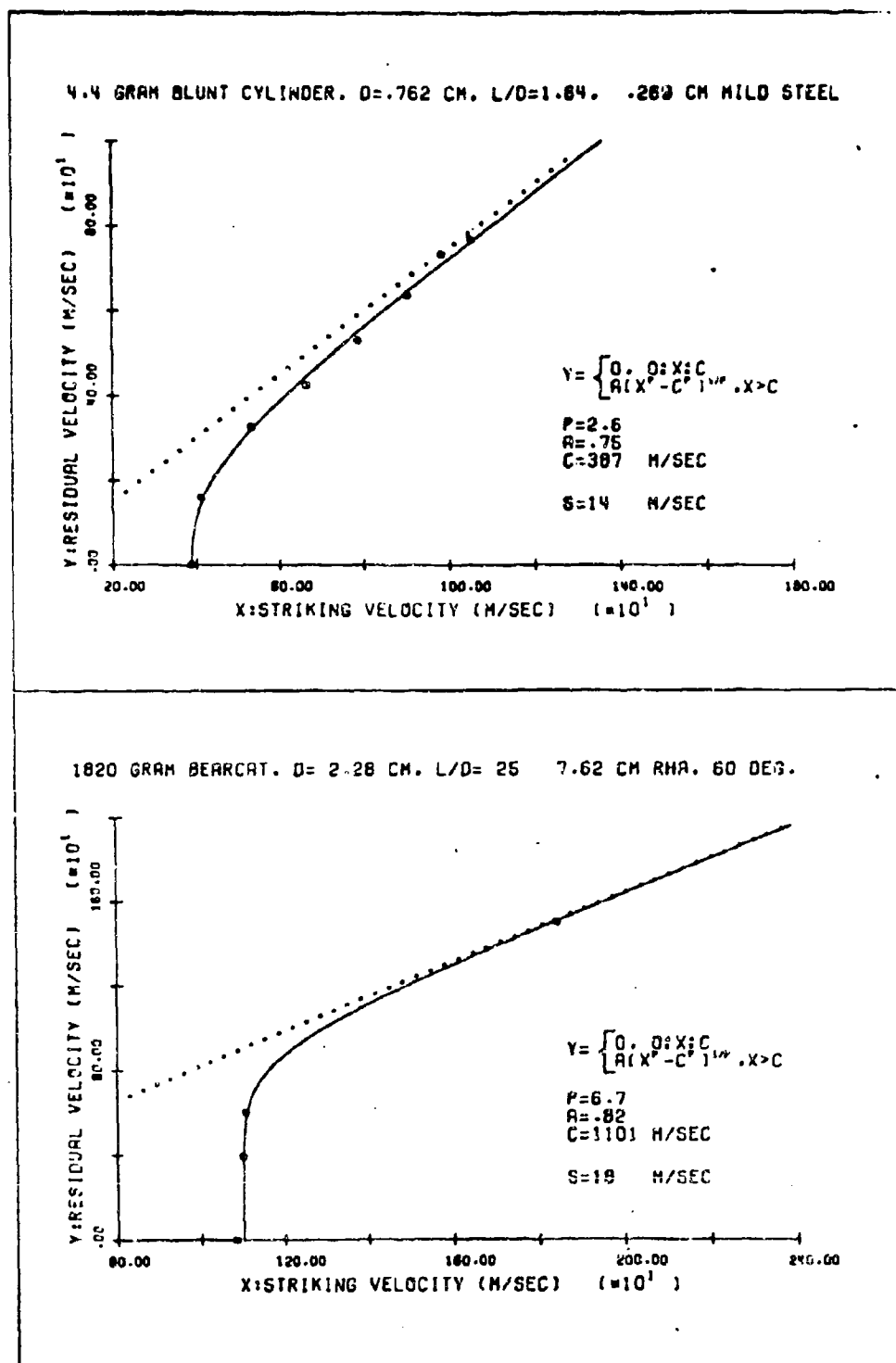


Figure 1. Two Examples of Generated  $v_s$  versus  $v_r$  Curves (Form (2) Representations)



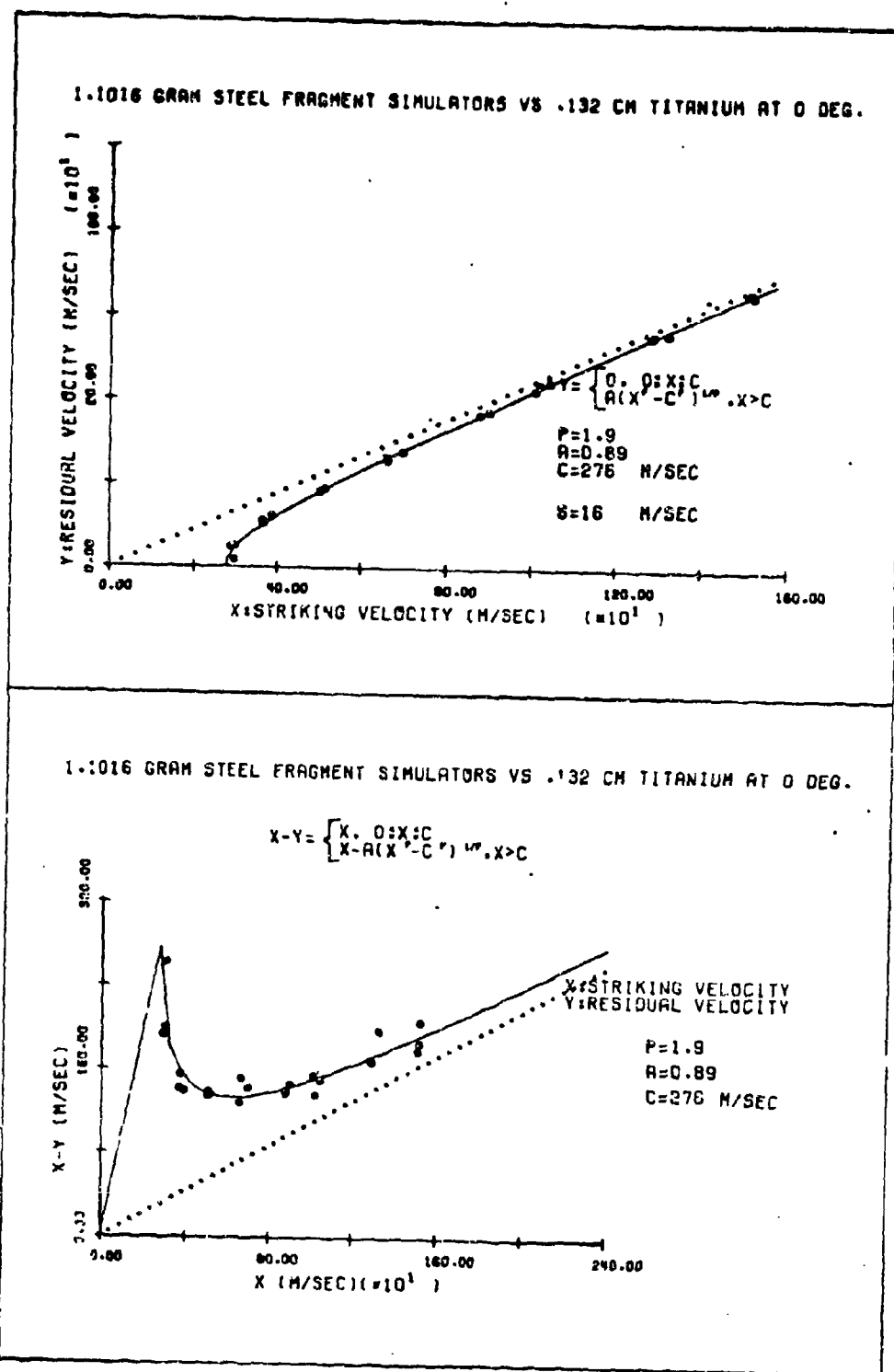


Figure 2. Example of  $v_s$  versus  $v_T$  Curve (Form (2)) and of Associated  $v_s$  versus  $v_s - v_T$  Curve (Form (3))

Table Ia. Sample of Computer Printout: First Page

27 GRAM BEARCAT STEEL ROD, D = .77 CM, L/D = 10, 1/2 IN. RHA, 0 DEG.

P = 3.4, A = .91, C = 802. M/SEC, AVG(Y-Y') = 1.  
 $Y' = A(X - P - C) * (1/P)$  IF  $X > C$ ,  $Y' = 0$  OTHERWISE

X	Y	Y'	Y-Y'	X-Y	X-Y'
746.	.	.	.	746.	746.
765.	.	.	.	765.	765.
796.	.	.	.	796.	796.
802.	.	.	.	802.	802.
807.	388.	229.	159.	418.	577.
817.	290.	326.	-37.	527.	491.
825.	394.	373.	21.	431.	452.
828.	386.	388.	-3.	443.	440.
833.	336.	407.	-71.	497.	426.
834.	352.	410.	-58.	482.	423.
867.	502.	515.	-13.	365.	352.
921.	621.	629.	-9.	301.	292.
930.	657.	644.	13.	272.	285.
1015.	769.	776.	-7.	247.	239.
1091.	871.	874.	-3.	219.	216.
1210.	1031.	1014.	17.	180.	197.
1303.	1128.	1114.	15.	174.	189.

X, Y AND Y' ARE IN METERS/SEC

Table Ib. Sample of Computer Printout: Second Page

27 GRAM BEARCAT STEEL ROD,  $\phi = .77$  CM,  $L/D = 10$ ,  $1/2$  IN. RHA, 0 DEG.

X	Y	X	Y	X	Y	X	Y
746.	.	765.	.	796.	.	802.	.
807.	388.	817.	290.	825.	394.	828.	386.
833.	336.	834.	352.	867.	502.	921.	621.
930.	657.	1015.	769.	1091.	871.	1210.	1031.
1303.	1128.						
P = 2.00	A = 1.000	C = 765.	S = 109.978				
P = 2.10	A = 1.000	C = 774.	S = 102.581				
P = 2.20	A = 1.000	C = 783.	S = 95.093				
P = 2.30	A = 1.000	C = 791.	S = 87.054				
P = 2.40	A = 1.000	C = 796.	S = 77.189				
P = 2.50	A = 1.000	C = 796.	S = 69.488				
P = 2.60	A = 1.000	C = 796.	S = 64.910				
P = 2.70	A = 1.000	C = 802.	S = 60.120				
P = 2.80	A = .995	C = 802.	S = 55.861				
P = 2.90	A = .978	C = 802.	S = 52.754				
P = 3.00	A = .962	C = 802.	S = 50.286				
P = 3.10	A = .947	C = 802.	S = 48.438				
P = 3.20	A = .934	C = 802.	S = 47.180				
P = 3.30	A = .922	C = 802.	S = 46.470				
P = 3.40	A = .910	C = 802.	S = 46.252				
P = 3.50	A = .900	C = 802.	S = 46.467				

27 GRAM BEARCAT STEEL ROD. D = .77 CM. L/D = 10. 1/2 IN. RHA. 0 DEG.

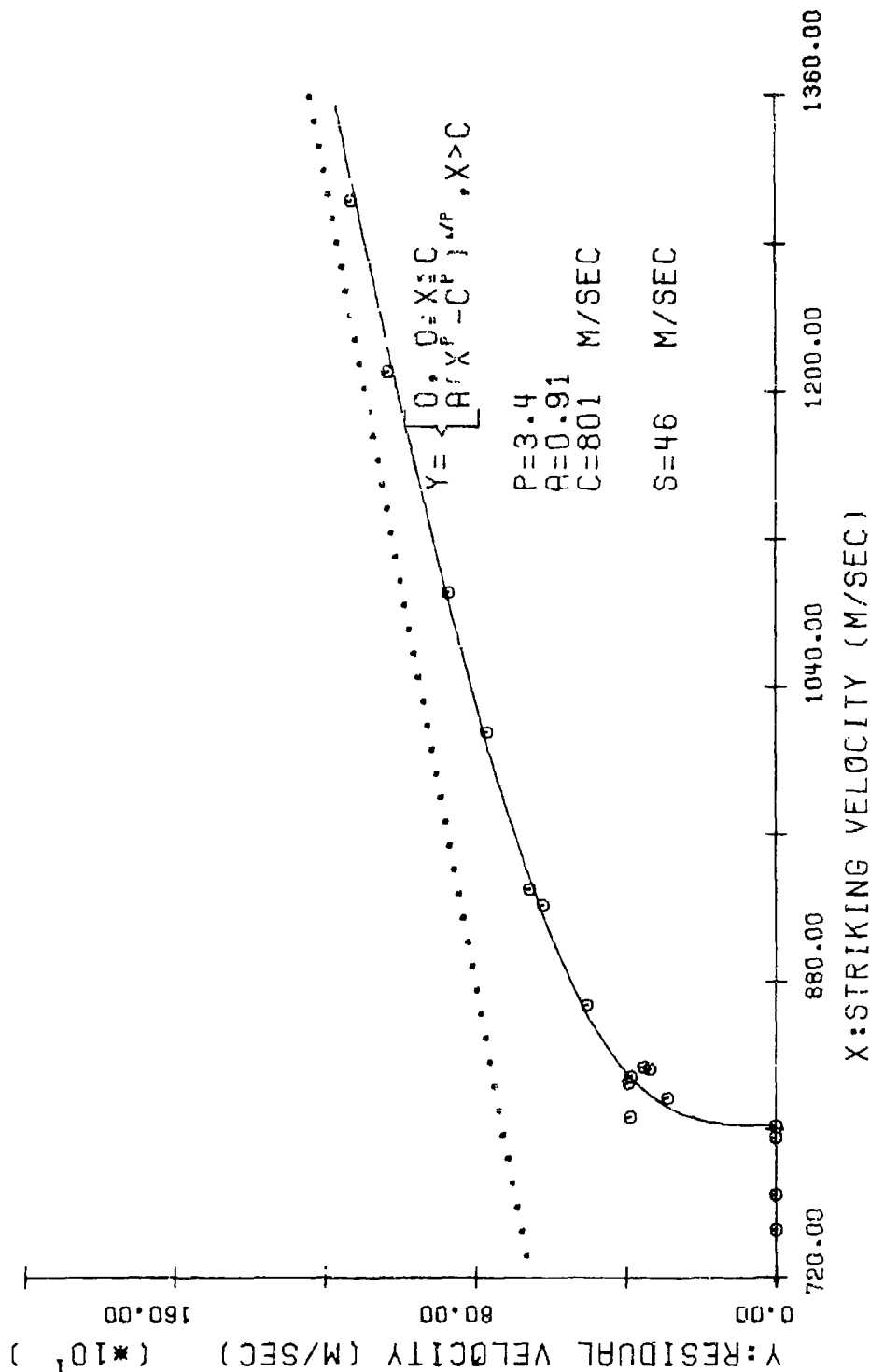


Figure 3a. Sample of Plotter Output:  $v_s$  versus  $v_r$

27 GRAM BEARCAT STEEL ROD, D = .77 CM, L/D = 10, 1/2 IN. RHA, 0 DEG.

$$X-Y = \begin{cases} X, & 0 \leq X \leq C \\ X-A(X^P-C^P), & X > C \end{cases}$$

X: STRIKING VELOCITY  
Y: RESIDUAL VELOCITY

P=3.4  
A=0.91  
C=801 M/SEC

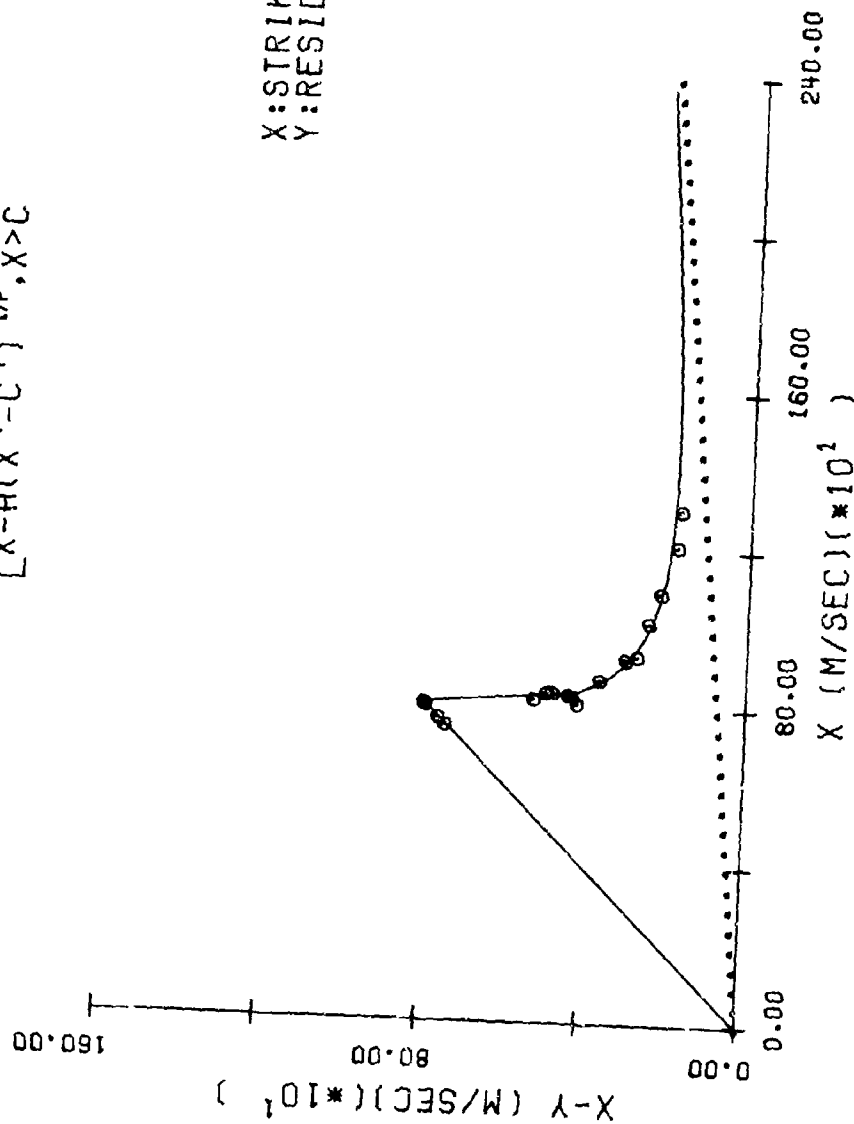


Figure 3b. Sample of Plotter Output:  $v_s$  versus  $v_s - v_r$

27 GRAM BEARCAT STEEL ROD, D = .77 CM, L/D = 10, 1/2 IN. RHA, 0 DEG.

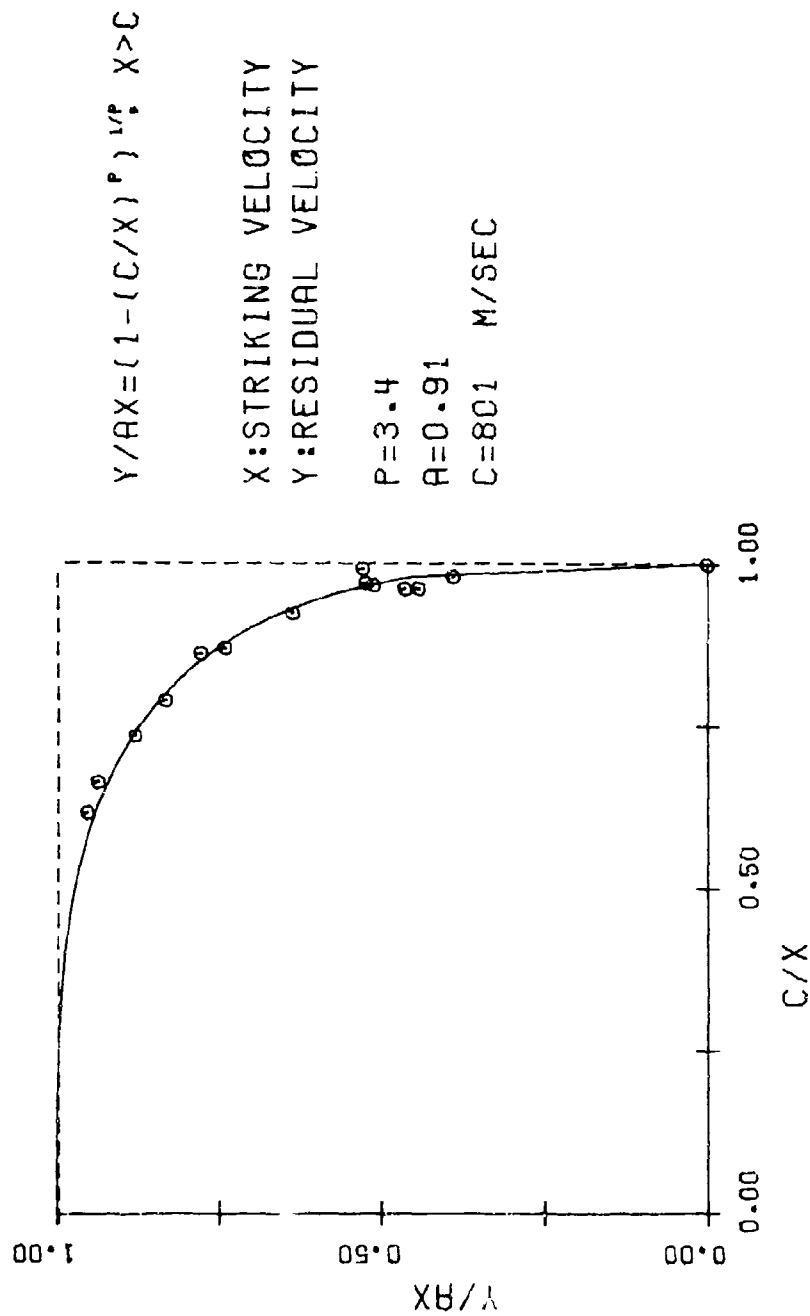


Figure 3c. Sample of Plotter Output:  $v_r/v_s$  versus  $v_r/\alpha v_s$

## 2. Input and Program

In Appendix A is found a guide to the input for running the regression procedure program listed in Appendix B. The program is written in Fortran language for BRLESC but could be adapted to other computers, with a few minor changes, i.e., replacements for SORTXY, SCOOP, length of Hollerith statements and USE (MAIN COMMON). SORTXY (X, Y, N) sorts the elements of vector X into non-decreasing order and moves the elements of Y so they correspond to the original values of X. SCOOP is the BRLESC implementation of the basic Fortran software package described in the publication "Programming CALCOMP Pen Plotters" [publication No. 1006A, California Computer Products, Anaheim, California, 1969]. A word of caution: XL, the third argument in PLOTS, is the length of paper required instead of the logical output device number as commonly used. This program is set up for 30 inch width paper.

Any or all the options and alternatives can be employed by setting the appropriate flag for each data set. The plots and tabulation will indicate which parameters, if any, are specified. In addition the first and third plots can have different scales. The usual procedure is for SCOOP to determine the initial X and Y values and data units/inch but this can be bypassed with the flag FIXAX and the alternative values stipulated.

The program employs 13K memory and a collection of sample problems averaging fifteen pairs of data points required an average of 19 seconds of computer time per problem.

## REFERENCES

1. M. Defourneaux, "Examination of the Theories Dealing with the Penetration of a Projectile into a Plastic Material," (Examen des Theories Relatives a la Penetration d'un Projectile dans un Matériau Plastique). Translation from the Army Foreign Service and Technology Center, Charlottesville, Virginia, FSTC-HT-23-0110-74, Sept. 1973.
2. W. Goldsmith and S. A. Finnegan, "Penetration and Perforation Processes in Metal Targets at and Above Ballistic Velocities," Int. J. Mech. Sci., 13, 1971.
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4. H. P. Robertson, "The Mechanics of Armor Perforation, I. Residual Velocity," National Defense Research Council, Armor and Ordnance Rept. No. A-227, OSRD No. 2043, Nov. 1943.



**APPENDIX A**

**INPUT DATA**

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# APPENDIX A

## INPUT DATA

CARD	FORMAT	COLUMNS	PROGRAM NAME	UNITS	REMARKS
1	F10.0	1-10	XL	inches	Maximum amount of plotting paper for this computer run. (Approximately 15 inches per set of 3 plots)
	F10.0	11-20	PINCR		Value used for incrementing parameter P. P=1 is used in this report
2	7A10	1-70	TITLE (1)		Title to head tables and graphs (up to 70 characters)
3	15	1-5	N		Number of data points. Dimension statements are set up for 97 but can be altered
	15	6-10	IFF		Dimension flag for (X, Y) data (m/sec=0, ft/sec=1)
	15	11-15	FIXAX		Axis flag for Plot 1 and Plot 3 (not specified=0, specified=1)
	15	16-20	PLT		Plots only, where P, A and C are given
	15	21-25	OPT(1)		Flag for parameter P given (yes=1, no=0)
	15	26-30	OPT(2)		Flag for parameter A given (yes=1, no=0)
	15	31-35	OPT(3)		Flag for parameters P and A given (yes=1, no=0)
	15	36-40	OPT(4)		Flag for eliminating standard program (yes=1, no=0)
4 etc.	8F10.0	1-80	X(1), Y(1)	ft/sec or m/sec	( $v_x$ , $v_y$ ) data, four pairs to a card, in units of m/sec or ft/sec as indicated by IFF flag

THE FOLLOWING CARDS ARE DEPENDENT ON ABOVE FLAGS: IF NONE, IGNORE

If FIXAX=1	F10.0	1-10	PLFX1	m/sec	Minimum X value for Plot 1 horizontal axis
	F10.0	11-20	PLDX1	m/sec	Number of data units per inch for Plot 1 horizontal axis
	F10.0	21-30	PLDY1	m/sec	Number of data units per inch for Plot 1 vertical axis which starts at zero
	F10.0	31-40	PLDX3	m/sec	Number of data units per inch for Plot 3 horizontal axis which starts at zero
	F10.0	41-50	PLDY3	m/sec	Number of data units per inch for Plot 3 vertical axis which starts at zero
If PLT=1	F10.0	1-10	C	m/sec	Parameter of equation
	F10.0	11-20	A		Parameter of equation
	F10.0	21-30	P		Parameter of equation
If OPT(1)=1 1=1, 3	F10.0	1-10	P		Parameter P given
	F10.0	11-20	A		Parameter A given

One card per option requested. For OPT (1) P only, OPT (2) A only, and OPT (3) both P and A

APPENDIX B

PROGRAM LISTING

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DIMENSION DS(100), DY(100), EY(100), OPT(4), VC(100)
COMMON A, AX(100), AY(100), BDY(5), BDZ(5), BX(7), BY(7),
1 CX(100), CY(100), FIXAX, FX(100), L, N, P, PINCR, PINV, S, SYSO.
2 TA(3), TB(3), TC(2), TD(2), TF(2), TO(3), TITLE(7), X(100),
3 Y(100), PLFX1, PLOX1, PLFY1, PLDY1, PLFX3, PLDX3, PLFY3, PLDY3,
4 PLT, IBUF(1000), XINC, YINC, TE, C, TG(2), TH(2), NCASE
INTEGER ALPHA, BETA, FIXAX, OPT, PLT
DATA (BDX(1),BDY(1),BDZ(1),I=1,5) /-1.2, -1.5, -1.8, 9.3, -1.5, -1
1.8, 9.3, 6.5, 6.2, -1.2, 6.5, 6.2, -1.2, -1.5, -1.8/
DATA (BX(1),BY(1),I=1,7) /5.87, 2.46, 5.76, 2.46, 5.76, 2.25, 5.69
1, 2.215, 5.76, 2.18, 5.76, 1.97, 5.87, 1.97/
DATA (TA(1),I=1,3) /10HX STRIKING, 10H VELOCITY, 10H(M/SEC) /
DATA (TB(1),I=1,3) /10HY/AX=(1-(C, 10H/X) ) , X>, 10HC /
DATA (TC(1),I=1,2) /10HAX -C ) , 10H,X>C /
DATA (TD(1),I=1,2) /10HP P , 10H1/P /
DATA TE /7H(GIVEN)/
DATA (TF(1),I=1,2) /10HX-AX -C ) , 10H ,X>C /
DATA (TG(1),I=1,2) /10HY-Y (M/SEC, 10H) /
DATA (TO(1),I=1,3) /10HY RESIDUAL, 10H VELOCITY, 10H(M/SEC) /
DATA (TH(1),I=1,2) /10HPARAMETERS, 10H GIVEN /
READ (5,37) XL,PINCR

C
C XL - TOTAL LENGTH OF PAPER NEEDED FOR GRAPHS
C PINCR - INCREMENT FOR P FOR OPTIONS 2 AND 4
C

CALL PLOTS (IBUF,1000,XL)
CALL PLOT (0,0,-36.0,-3)
CALL PLOT (5.0,2.0,-3)
XINC=15.
YINC=9.
FX(1)=0.
DO 1 I=2,50
1 FX(I)=FLOAT(I-1)*.02
FX(51)=1.0
FX(52)=0.0
FX(53)=.25
C
C *** TITLE WILL HEAD ALL GRAPHS AND TABULATIONS
2 READ (5,35) (TITLE(I),I=1,7)
WRITE (6,39) (TITLE(I),I=1,7)
IHEADG=0
READ (5,36) N,IFF,FIXAX,PLT,(OPT(I),I=1,4),(X(J),Y(J),J=1,N)

C
C N - NUMBER OF DATA PAIRS (DIMENSIONING ALLOWS MAXIMUM OF
C IFF - DIMENSION FLAG FOR DATA M/SEC=0 FT/SEC=1
C FIXAX - FIXED SCALE FOR AXIS NO=0 YES=1
C PLT - PLOT WITH GIVEN A,P,AND C YES=1 NO=0
C OPT(1) - P GIVEN YES=1 NO=0
C OPT(2) - A GIVEN YES=1 NO=0
C OPT(3) - A AND P GIVEN YES=1 NO=0
C OPT(4) - ELIMINATE STANDARD PROGRAM YES=1 NO=0
C X - STRIKING VELOCITY
C Y - RESIDUAL VELOCITY
C

IF (FIXAX.GT.0) READ (5,37) PLFX1,PLDX1,PLDY1,PLDX3,PLDY3

C
C PLFX1 - STARTING VALUE ON HORIZONTAL AXIS PLOT1
C PLOX1 - DATA UNITS PER INCH HORIZONTAL AXIS PLOT1
C PLDY1 - DATA UNITS PER INCH VERTICAL AXIS PLOT1
C PLOX3 - DATA UNITS PER INCH HORIZONTAL AXIS PLOT3

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C      PLDY3 - DATA UNITS PER INCH VERTICAL AXIS PLOT3
C
  IF (IFF.EQ.0) GO TO 4
  *** CONVERTS FEET TO METERS
  DO 3 K=1,N
  X(K)=X(K)*.3048
  Y(K)=Y(K)*.3048
  C      *** ARRANGING DATA IN INCREASING ORDER FOR STRIKING VELOCITIES
  4 CALL SORTX (X,Y,N)
  SYSQ=0.
  DO 5 I=1,N
  5 SYSQ=SYSQ+Y(I)*Y(I)
  IF (PLT.GT.0) READ (5,37) C,A,P
C
C      C - LIMIT VELOCITY
C      A - SLOPE
C      P - EXPONENT OF FIT
C
  WRITE (6,38) (X(I),Y(I),I=1,N)
  6 DO 32 L=1,4
  IF (PLT.GT.0) GO TO 25
  IF (OPT(L).EQ.0.AND.L.LT.4) GO TO 32
  IF (L.EQ.4.AND.OPT(4).EQ.1) GO TO 32
  IF (L.EQ.4) GO TO 8
  JFIN=0
  JOPT=1
  7 IF (OPT(L).LT.JOPT) GO TO 32
  READ (5,37) P,A
C
C      P,A - PRESCRIBED VALUES. NONE TO THREE DATA CARDS DEPEND-
C      ING ON OPTIONS
C
  JOPT=JOPT+1
  GO TO 9
  8 JFIN=1
  9 DO 10 I=1,N
  ALPHA=I
  IF (Y(I).GT.0.) GO TO 11
  10 CONTINUE
  11 BETA=0
  NM1=N-1
  DO 12 I=1,NM1
  IF (Y(I).EQ.0.) BETA=I
  12 CONTINUE
  C      *** INITIAL ESTIMATE OF C
  B1=0.
  B2=0.
  B3=0.
  B4=0.
  IF (L.EQ.2.OR.L.EQ.4) P=2.
  DO 13 K=1,N
  IF (Y(K).GT.0.) GO TO 14
  13 CONTINUE
  14 DO 15 I=K,N
  XP=X(I)**P
  YP=Y(I)**P
  B1=B1+XP**2
  B2=B2+XP
  B3=B3+YP
  15 B4=B4+XP*YP

```

```

R=N-K+1
RR1=R*B1
B2SQ=B2*B2
IF (RB1.EQ.B2SQ) GO TO 16
U=(R*B4-B2*B3)/(RB1-B2SQ)
GO TO 17
16 U=0.
17 IF (U.LE.0.) GO TO 18
T=(B2*U-B3)/(R*U)
IF (T.LE.0.) GO TO 18
T=T**(.1./P)
GO TO 19
18 T=0.
19 TI=AIN(T)
TK=T-TI
T=TI
IF (TK.GT..5) T=TI+1.
IF (ALPHA.LE.BETA) CZ=AMAX1(T,0.)
IF (ALPHA.GT.BETA) CZ=AMAX1(T,X(BETA))
C=CZ
WRITE (6,40)
II=0
IF (IHEADG.EQ.1) GO TO 34
IHEADG=1
20 CALL PARAMETERS
WRITE (6,41) P,A,C,S
IF (L.EQ.2.OR.L.EQ.4) GO TO 26
21 WRITE (6,42)
C **** SUMMARY TABLE OF DATA FOR PLOTS
AVG=0.
PINV=1./P
DO 24 I=1,N
IF (X(I).LE.C) GO TO 23
YC(I)=A*(X(I)**P-C**P)**PINV
22 EY(I)=Y(I)-YC(I)
DS(I)=X(I)-YC(I)
DY(I)=X(I)-Y(I)
AVG=AVG+EY(I)
GO TO 24
23 YC(I)=0.
GO TO 22
24 CONTINUE
AVG=AVG/FLOAT(N)
WRITE (6,43) (TITLE(I),I=1,7),P,A,C,AVG
WRITE (6,44) (X(I),Y(I),YC(I),EY(I),DY(I),DS(I),I=1,N)
WRITE (6,45)
IF (L.EQ.1) WRITE (6,46)
IF (L.EQ.2) WRITE (6,47)
IF (L.EQ.3) WRITE (6,48)
WRITE (6,42)
25 CONTINUE
IF (PLT.GT.0) L=0
C **** PLOT * VS Y (STRIKING VELOCITY VS RESIDUAL VELOCITY)
CALL PLOT1
C **** PLOT C/X VERSUS Y/AX
CALL PLOT2
C **** PLOT X VS X-Y
CALL PLOT3
IF (PLT.GT.0) GO TO 33
IF (JFIN.EQ.0) GO TO 7

```

GO TO 32  
C \*\*\*\* DETERMINATION OF P

```

26 IF (11.GT.0) GO TO 28
   II=1
   DIR=PINCR
   MM=0
27 PREVA=A
   PREVC=C
   PREVS=S
   PREVP=P
   P=P+DIR
   GO TO 20
28 IF (S.GE.PREVS) GO TO 30
   IF (P.GE.8.0.OR.P.LE.1.1) GO TO 21
29 MM=1
   GO TO 27
30 IF (MM.GT.0) GO TO 31
   DIR=-PINCR
   GO TO 29
31 C=PREVC
   A=PREVA
   P=PREVP
   S=PREVS
   GO TO 21
32 CONTINUE
   GO TO 2
33 PLT=0
   IHEADG=1
   GO TO 6
34 WRITE (5,39) (TITLE(I),I=1,7)
   WRITE (6,38) (X(I),Y(I),I=1,N)
   GO TO 20

```

C

```

35 FORMAT (7A10)
36 FORMAT (8I5/(8F10.0))
37 FORMAT (8F10.0)
38 FORMAT (///6X,1HX,9X,1HY,9X,1HX,9X,1HY,9X,1HX,9X,1HY,9X,1HX,9X,1HY/
  1/(3X,F6.0,4X,F6.0,3X,F6.0,4X,F6.0,4X,F6.0,4X,F6.0,4X,F6.0,4X,F6.0)
  2)
39 FORMAT (1X,7A10)
40 FORMAT (///)
41 FORMAT ('P=',F5.2,3X,'A=',F6.3,3X,'C=',F6.0,3X,'S=',F9.3)
42 FORMAT (1H1)
43 FORMAT (///,1GX,7A10///15X,2HP=,F4.1,5H, A=,F5.2,5H, C=,F6.0,18H
  1 M/SEC, AVG(Y-Y')=,F6.0/15X,49HY'=A(X**P-C**P)**(1/P) IF X>C, Y'=
  20 OTHERWISE ///17X,1HX,8X,1HY,8X,2HY',6X,4HY-Y',5X,3HX-Y,6X,4HX-
  3Y')
44 FORMAT (14X,F6.0,3X,F6.0,3X,F6.0,3X,F6.0,3X,F6.0,3X,F6.0)
45 FORMAT (///1GX,28HX,Y AND Y' ARE IN METERS/SEC,///)
46 FORMAT (10X,'PGIVEN')
47 FORMAT (10X,'AGIVEN')
48 FORMAT (10X,'PANDAGIVEN')
   END

```

C  
C  
C

```

SUBROUTINE PARAMETERS
COMMON (USE MAIN)
DIMENSION XP(100), YP(100)
M=0

```

```

C      **** DETERMINATION OF A
      DO 1 K=1,N
      XP(K)=X(K)**P
1     YP(K)=Y(K)**P
      PINV=1./P
2     IF (C.LT.0.) GO TO 9
      CP=C**P
      DO 3 K=1,N
      IF (X(K).GE.C) GO TO 4
3     CONTINUE
4     G=0.
      H=0.
      DO 5 I=K,N
      XPMCP=(XP(I)-CP)**PINV
      G=G+Y(I)*XPMCP
5     H=H+XPMCP**2
      GDH=G/H
      IF (L.EQ.1.OR.L.EQ.4) A=AMIN1(GDH,1.)
      PHI=A*(2.*G-A*H)

```

```

C      ** DETERMINATION OF C AND S
      IF (H.GT.0) GO TO 7
      M=1
      DIR=1.
      MM=0
6     SAVEA=A
      SAVEP=PHI
      SAVEC=C
      SAVEG=G
      SAVEH=H
      C=C+DIR
      GO TO 2
7     IF (PHI.GE.SAVEP) GO TO 8
      IF (MM.GT.0) GO TO 9
      DIR=-1.
8     MM=1
      GO TO 6
9     A=SAVEA
      C=SAVEC
      PSI=ABS(SYSQ-A*(2.*SAVEG-A*SAVEH))
      S=SQRT(PHI/FLOAT(N))
      RETURN

```

```

C      END

```

```

C
C
C

```

```

      SUBROUTINE PLOT1
      COMMON (USE MAIN)
      JJ=0
      PINV=1./P
      IF (FIXAX.GT.0) GO TO 16
      IF (C.LT.X(1)) GO TO 13
1     CALL SCALE (X,8.0,N,1)
C     ASSURES Y AXIS STARTS AT ZERO
      Y(N+1)=0.
      N=N+1
      CALL SCALE (Y,5.0,N,1)
      Y(N)=Y(N+1)
      Y(N+1)=Y(N+2)
      N=N-1

```



```

      IF (JJ.EQ.1) GO TO 14
2  AA=C-X(N+2)
      IF (AA.LT.X(N+1)) GO TO 15
C  *** PLOT TITLES, AXES, AND PARAMETERS
3  CALL AXIS (0.0,0.0,TA,-30,8.0,0.0,C,X(N+1),X(N+2))
      CALL AXIS (0.0,0.0,TO,30,5.0,90.0,Y(N+1),Y(N+2))
      IF (FIXAX.GT.0) GO TO 18
      CALL LINE (X,Y,N,1,-1,1)
4  CALL SYMBOL (-.5,5.5,.14,TITLE,0.0,70)
      CALL SYMBOL (6.46,2.32,.07,5HC<,0.0,5)
      CALL SYMBOL (5.9,2.25,.14,8HO, 0 X C,0.0,8)
      CALL SYMBOL (6.46,2.25,.07,5H=,0.0,5)
      CALL SYMBOL (5.34,2.18,.14,2HY=,0.0,2)
      CALL SYMBOL (6.32,2.145,.07,TD,0.0,20)
      CALL SYMBOL (5.9,2.04,.14,TC,0.0,20)
      CALL SYMBOL (5.34,1.59,.14,2HP=,0.0,2)
      CALL NUMBER (5.62,1.59,.14,P,C,0,1)
      IF (L.EQ.1.OR.L.EQ.3) CALL SYMBOL (6.32,1.59,.14,TE,0.0,7)
      CALL SYMBOL (5.34,1.38,.14,2HA=,0.0,2)
      CALL NUMBER (5.62,1.38,.14,A,C,0,2)
      IF (L.EQ.2.OR.L.EQ.3) CALL SYMBOL (6.32,1.38,.14,TE,0.0,7)
      CALL SYMBOL (5.34,1.17,.14,2HC=,0.0,2)
      CALL NUMBER (5.62,1.17,.14,C,C,0,-1)
      CALL SYMBOL (6.32,1.17,.14,5HM/SEC,0.0,5)
      IF (PLT.EQ.0) CALL SYMBOL (5.34,.66,.14,2HS=,0.0,2)
      IF (PLT.EQ.0) CALL NUMBER (5.62,.66,.14,S,0.0,-1)
      IF (PLT.EQ.0) CALL SYMBOL (6.32,.66,.14,5HM/SEC,0.0,5)
      IF (PLT.GT.0) CALL SYMBOL (5.34,.66,.14,TH,0.0,20)
C  *** PLOT BRACKETS
      CX(8)=X(N+1)
      CX(9)=X(N+2)
      CY(8)=Y(N+1)
      CY(9)=Y(N+2)
      DO 5 I=1,7
      CX(I)=8X(I)*CX(9)+CX(8)
5  CY(I)=8Y(I)*CY(9)+CY(8)
      CALL LINE (CX,CY,7,1,0,1)
C  *** PLOT CURVE
      XMAX=X(N+1)+8.*X(N+2)
      YMAX=Y(N+1)+5.*Y(N+2)
      NN=2
      AX(1)=X(N+1)
      AY(1)=0.0
      AX(2)=C
      AY(2)=0.0
      DR=(Y(N+1)+5.0*Y(N+2))/48.
      IF (AX(2).LT.AX(1)) GO TO 17
6  DO 7 I=3,50
      AY(I)=AY(I-1)+DR
      IF (AY(I).GE.YMAX) GO TO 8
      AX(I)=((AY(I)/A)**P+C**P)**PINV
      IF (AX(I).GE.XMAX) GO TO 8
      NN=I
7  CONTINUE
8  AX(NN+1)=X(N+1)
      AX(NN+2)=X(N+2)
      AY(NN+1)=Y(N+1)
      AY(NN+2)=Y(N+2)
      CALL LINE (AX,AY,NN,1,0,0)
C  *** PLOT ASYMPTOTE

```

```

NN=1
AX(1)=X(N+1)
AY(1)=Y(N+1)
DR=(XMAX-X(N+1))/49.
DO 9 I=2,50
  AX(I)=AX(I-1)+DR
  IF (AX(I).GE.XMAX) GO TO 10
  AY(I)=AX(I)*A
  IF (AY(I).GE.YMAX) GO TO 10
9 NN=1
10 DO 11 I=1,NN
  AX(I)=(AX(I)-X(N+1))/X(N+2)-.07
  AY(I)=(AY(I)-Y(N+1))/Y(N+2)-.035
11 CALL SYMBOL (AX(I),AY(I),.14,1H.,0.0,1)
C   *** PLOT BORDERS
  CX(6)=X(N+1)
  CX(7)=X(N+2)
  CY(6)=Y(N+1)
  CY(7)=Y(N+2)
  DO 12 I=1,5
    CX(I)=BDX(I)*CX(7)+CX(6)
    CY(I)=BDY(I)*CY(7)+CY(6)
12 CALL LINE (CX,CY,5,1,0,1)
  CALL PLOT (0.,YINC,-3)
  WRITE (6,20)
  RETURN
13 SAVE1=X(1)
  SAVE2=Y(1)
  JJ=1
  X(1)=C
  Y(1)=0.
  GO TO 1
14 X(1)=SAVE1
  Y(1)=SAVE2
  GO TO 2
C   ASSURES SPACE ON PLOT BETWEEN XMIN AND X=C
15 X(N+1)=AA
  Y(N+1)=0.
  N=N+1
  IF (X(N).LE.0.) X(N)=0.
  CALL SCALE (X,8.0,N,1)
  CALL SCALE (Y,5.0,N,1)
  X(N)=X(N+1)
  Y(N)=Y(N+1)
  X(N+1)=X(N+2)
  Y(N+1)=Y(N+2)
  N=N-1
  GO TO 3
16 X(N+1)=PLFX1
  X(N+2)=PLDX1
  Y(N+1)=0.
  Y(N+2)=PLY1
  GO TO 3
17 AY(2)=A*(AX(1)**P-C**P)**PINV
  AX(2)=AX(1)
  GO TO 6
18 K=0
  DO 19 J=1,N
    IF (X(J).LT.PLFX1) GO TO 19
    K=K+1

```

```

      AX(K)=X(J)
      AY(K)=Y(J)
19  CONTINUE
      AX(K+1)=X(N+1)
      AX(K+2)=X(N+2)
      AY(K+1)=Y(N+1)
      AY(K+2)=Y(N+2)
      CALL LINE (AX,AY,K,1,-1,1)
      GO TO 4
C
20  FORMAT (12H PLOT 1 DONE)
      END
C
C
C
      SUBROUTINE PLOT2
      COMMON (USE MAIN)
      DIMENSION DL(3)
      DATA (DL(I),I=1,3) /10H-----, 10H-----, 10H----- /
C      *** PLOT TITLES, AXES, AND PARAMETERS
      CALL AXIS (0.0,0.0,3HC/X,-3,4.0,0.0,0.0,.25)
      CALL AXIS (0.0,0.0,4HY/AX,4,4.0,90.0,0.0,.25)
      CALL SYMBOL (-.06,3.93,.14,DL,0.0,30)
      CALL SYMBOL (4.07,-.06,.14,DL,90.0,30)
      CALL SYMBOL (-.5,4.5,.14,TITLE,0.0,70)
      CALL SYMBOL (6.355,3.605,.07,7HP 1/P,0.0,7)
      CALL SYMBOL (4.5,3.5,.14,TB,0.0,22)
      CALL SYMBOL (4.5,2.7,.14,TA,0.0,19)
      CALL SYMBOL (4.5,2.4,.14,TO,0.0,19)
      CALL SYMBOL (4.5,1.9,.14,2HP=,0.0,2)
      CALL NUMBER (4.78,1.9,.14,P,0.0,1)
      IF (L.EQ.1.OR.L.EQ.3) CALL SYMBOL (5.48,1.9,.14,TE,0.0,7)
      CALL SYMBOL (4.5,1.6,.14,2HA=,0.0,2)
      CALL NUMBER (4.78,1.6,.14,A,0.0,2)
      IF (L.EQ.2.OR.L.EQ.3) CALL SYMBOL (5.48,1.6,.14,TE,0.0,7)
      CALL SYMBOL (4.5,1.3,.14,2HC=,0.0,2)
      CALL NUMBER (4.78,1.3,.14,C,0.0,-1)
      CALL SYMBOL (5.48,1.3,.14,5HM/SEC,0.0,5)
      IF (PLT.GT.0) CALL SYMBOL (4.50,.79,.14,TH,0.0,20)
C      *** PLOT CURVE
      AX(1)=1.
      PINV=1./P
      DO 1 I=2,50
1  AX(I)=(1.-FX(I)**P)**PINV
      AX(51)=0.0
      AX(52)=0.0
      AX(53)=.25
      CALL LINE (FX,AX,51,1,0,0)
C      *** PLOT DATA
      J=0
      DO 2 I=1,N
      IF (X(I).LT.C) GO TO 2
      J=J+1
      CX(J)=C/X(I)
      CY(J)=Y(I)/(A*X(I))
2  CONTINUE
      CX(J+1)=0.0
      CX(J+2)=.25
      CY(J+1)=0.0
      CY(J+2)=.25

```

```

C      CALL LINE (CX,CY,J,1,-1,1)
      *** PLOT BORDERS
      CX(6)=0.0
      CX(7)=.25
      CY(6)=0.0
      CY(7)=.25
      DO 3 I=1,5
      CX(I)=80X(I)*CX(7)+CX(6)
3  CY(I)=80Z(I)*CY(7)+CY(6)
      CALL LINE (CX,CY,5,1,0,1)
      CALL PLOT (0.,YINC,-3)
      WRITE (6,4)
      RETURN

C
4  FORMAT (12H PLOT 2 DONE)
   END

C
C
C
      SUBROUTINE PLOT3
      COMMON (USE MAIN)
      DIMENSION DY(100)
      DO 1 I=1,N
1  DY(I)=X(I)-Y(I)
      IF (FIXAX.GT.0) GO TO 10
      X(N+2)=C
      DY(N+2)=C
      X(N+1)=0.
      DY(N+1)=0.
      N=N+2
      CALL SCALE (X,6.0,N,1)
      CALL SCALE (DY,4.0,N,1)
      X(N-1)=X(N+1)
      X(N)=X(N+2)
      DY(N-1)=DY(N+1)
      DY(N)=DY(N+2)
      N=N-2
C      *** PLOT TITLES, AXES, AND PARAMETERS
2  CALL AXIS (0.0,0.0,9HX (M/SEC),-9.6,0.0,0.0,X(N+1),X(N+2))
      CALL AXIS (0.0,0.0,TG,11.4,3,90.0,DY(N+1),DY(N+2))
      CALL SYMBOL (-.5,5.5,.14,TITLE,0.0,70)
      CALL SYMBOL (1.98,4.72,.14,4HX-Y=,0.0,4)
      CALL SYMBOL (2.82,4.79,.14,8HX, 0 X C,0.0,8)
      CALL SYMBOL (3.41,4.86,.07,5H<  <,0.0,5)
      CALL SYMBOL (3.41,4.79,.07,5H=  =,0.0,5)
      CALL SYMBOL (3.59,4.685,.07,TD,0.0,20)
      CALL SYMBOL (2.82,4.58,.14,TF,0.0,20)
      CALL SYMBOL (5.52,3.0,.14,TA,0.0,19)
      CALL SYMBOL (5.52,2.8,.14,TG,0.0,19)
      CALL SYMBOL (6.5,2.3,.14,2HP=,0.0,2)
      CALL NUMBER (6.78,2.3,.14,P,0.0,1)
      IF (L.EQ.1.OR.L.EQ.3) CALL SYMBOL (7.48,2.3,.14,TE,0.0,7)
      CALL SYMBOL (6.5,2.0,.14,2HA=,0.0,2)
      CALL NUMBER (6.78,2.0,.14,A,0.0,2)
      IF (L.EQ.2.OR.L.EQ.3) CALL SYMBOL (7.48,2.0,.14,TE,0.0,7)
      CALL SYMBOL (6.5,1.7,.14,2HC=,0.0,2)
      CALL NUMBER (6.78,1.7,.14,C,0.0,-1)
      CALL SYMBOL (7.34,1.7,.14,6HM/SEC ,0.0,6)
      IF (PLT.GT.0) CALL SYMBOL (6.5,1.19,.14,TH,0.0,20)
C      *** PLOT BRACKETS

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```

CX(8)=X(N+1)
CX(9)=X(N+2)
CY(8)=DY(N+1)
CY(9)=DY(N+2)
DO 3 I=1,7
CX(I)=(BX(I)-3.08)*CX(9)+CX(8)
3 CY(I)=(BY(I)+2.54)*CY(9)+CY(8)
CALL LINE (CX,CY,7,1,0,1)
C   *** PLOT CURVE
NN=2
CX(1)=0.
CY(1)=0.
CX(2)=C
CY(2)=C
RMAX=4.*DY(N+2)
ZMAX=6.*X(N+2)
DZ=(ZMAX-CX(2))/48.
PINV=1./P
DO 4 I=3,50
CX(I)=CX(I-1)+DZ
IF (CX(I).GT.ZMAX) GO TO 5
CY(I)=CX(I)-A*(CX(I)**P-C**P)**PINV
IF (CY(I).GT.RMAX) GO TO 5
NN=I
4 CONTINUE
5 CX(NN+1)=X(N+1)
CX(NN+2)=X(N+2)
CY(NN+1)=DY(N+1)
CY(NN+2)=DY(N+2)
CALL LINE (CX,CY,NN,1,0,0)
C   *** PLOT ASYMPTOTE
D=(1.-A)*ZMAX
IF (D.GT.RMAX) GO TO 11
6 Z=ZMAX/FLOAT(NN-1)
CX(1)=0.
CY(1)=0.
DO 7 I=2,NN
CX(I)=CX(I-1)+Z
7 CY(I)=(1.-A)*CX(I)
DO 8 I=1,NN
CX(I)=(CX(I)-X(N+1))/X(N+2)-.07
CY(I)=(CY(I)-DY(N+1))/DY(N+2)-.035
CALL SYMBOL (CX(I),CY(I),.14,1H.,0.0,1)
8 CONTINUE
C   *** PLOT DATA
CALL LINE (X,DY,N,1,-1,1)
C   *** PLOT BORDERS
CX(6)=X(N+1)
CX(7)=X(N+2)
CY(6)=Y(N+1)
CY(7)=Y(N+2)
DO 9 I=1,5
CX(I)=BDX(I)*CX(7)+CX(6)
9 CY(I)=BDY(I)*CY(7)+CY(6)
CALL LINE (CX,CY,5,1,0,1)
YORIG=-2.*YINC
CALL PLOT (XINC,YORIG,-3)
WRITE (6,12)
WRITE (6,13)
RETURN

```

```

      RETURN
10  X(N+1)=0.
    X(N+2)=PLOX3
    DY(N+1)=0.
    DY(N+2)=PLDY3
    GO TO 2
11  ZMAX=RMAX/(1.-A)
    GO TO 6
C
12  FORMAT (12H PLOT 3 DONE)
13  FORMAT (1H1)
    END

```

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